Physics 541 Compact Objects

Final Exam

Instructions.

You are permitted to use any books, notes, reference material, or calculation tools that you require, but you may not consult with anyone else while taking the exam. If any significant correlation is detected between your exam paper and someone else's, you will be invited in for an oral exam.

The exam is **due Thursday May 12, 2005 at noon** in the homework box. The exams will be picked up precisely at noon. Late exams will not be accepted.

If you have questions regarding the final, please direct them to `gammie@uiuc.edu`.

1. Consider circular orbits in the Schwarzschild geometry, whose line element is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -Adt^2 + A^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2,$$

where $A \equiv 1 - 2GM/(rc^2)$. The orbital frequency as seen by a distant observer is

$$\Omega = \frac{u^\phi}{u^t} = (GM)^{1/2} r^{-3/2}.$$

Find a general expression for the energy per unit rest mass of a circular, equatorial orbit and evaluate it at $r = 6GM/c^2$. [15 points]

2. Consider a nonrotating $10\,\mathrm{M}_\odot$ black hole accreting through a thin disk at one half of the Eddington luminosity. Use α disk theory, where appropriate, to estimate answers to the following questions. Assume $\alpha = 0.1$.

a. What is the angular size of the black hole at a distance of 1kpc? Will gravitational lensing make the black hole event horizon appear larger or smaller on the sky? Give your answer in seconds of arc. [5 points]

b. What is the accretion rate? [5 points]

c. How thick is the disk at $10GM/c^2$? [5 points]

d. What is the effective (surface) temperature of the disk at $10GM/c^2$? [5 points]

e. Estimate the magnetic field strength in the disk at $10GM/c^2$. State your assumptions. [5 points]

f. What is the wavelength of peak emission from the disk, assuming that it radiates locally like a blackbody? [5 points]

g. Holding the accretion rate constant, would the luminosity increase or decrease if the black hole were rotating? [5 points]

3. Consider a neutron star in a molecular cloud of density 10^3 hydrogen molecules cm^{-3} and temperature 20K. You may assume that γ, the ratio of specific heats, is 7/5 (this is not actually a good assumption).

a. Using Bondi-Hoyle-Lyttleton accretion theory, estimate the accretion rate onto the neutron star. [5 points]

b. What is the luminosity of the accreting neutron star? Explain your reasoning. [5 points]

c. Would a neutron star in a molecular cloud actually be able to radiate at this luminosity? Comment on physical processes that may have been left out of our analysis. [5 points]

d. (unrelated to c.) What would the luminosity be if the neutron star were moving at 10 km s^{-1}? [5 points]

4. Consider a nonrotating white dwarf composed of ^{4}He. What is its maximum gravitational redshift? [15 points]

5. Find the central pressure of an $0.6\,M_\odot$ white dwarf composed of ^{4}He. [15 points]

6. A pulsar is discovered with a period of 20 milliseconds and a spin-down age of 2000 years. What is the magnetic field strength on the pulsar's surface? State your assumptions clearly. [10 points]